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(Dated: December 3rd 2007)

The hadronic $k_\perp$-spectrum inside one jet is determined including corrections of relative magnitude $\mathcal{O}\left(\sqrt{\alpha_s}\right)$ with respect to the Modified Leading Logarithmic Approximation (MLLA), at and beyond the limiting spectrum (assuming an infrared cut-off $Q_0 = \Lambda_{_{\rm QCD}}$ and $Q_0 \neq \Lambda_{_{\rm QCD}}$). The agreement between our results and preliminary measurements by the CDF collaboration is impressive, much better than at MLLA, pointing out very small overall non-perturbative contributions.

PACS numbers: 12.38.Cy, 13.87.-a., 13.87.Fh

Jet production – a collimated bunch of hadrons – in e^+e^-, e^-p and hadronic collisions is an ideal playground for parton evolution in perturbative QCD (pQCD). One of the major successes of pQCD is the hump-backed shape of inclusive spectra, predicted in [1] within MLLA, and later discovered experimentally (see e.g. [2]). Refining the comparison of pQCD calculations with data taken at LEP, Tevatron and LHC will ultimately allow for a crucial test of the Local Parton Hadron Duality (LPHD) hypothesis [3] and for a better understanding of color neutralization processes. In this Letter, a class of next-to-next-to-leading logarithmic (NMLLA) corrections to the single inclusive $k_\perp$-distribution of hadrons inside one jet is determined. Unlike other NMLLA corrections, these terms better account for recoil effects and were shown to drastically affect multiplicities and particle correlations in jets [4]. We start by writing the MLLA evolution equations for the fragmentation function $D_B^h\left(x/z, zE\Theta_0, Q_0\right)$ of a parton B (energy zE and transverse momentum $k_\perp = zE\Theta_0$) into a gluon (identified as a hadron h with energy xE according to LPHD) inside a jet of energy E. As a consequence of angular ordering in parton cascading, partonic distributions inside a quark and gluon jet, $Q, G(z) = x/z\ D_{Q,G}^h\left(x/z, zE\Theta_0, Q_0\right)$, obey the system of two coupled equations [5] (the subscript $_y$ denotes $\partial/\partial y$)

$$Q_y = \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{\pi}\,\Phi_q^g(z)\left[\Big(Q(1-z)-Q\Big)+G(z)\right], \quad (1)$$

$$G_y = \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{\pi}\left[\Phi_g^g(z)(1-z)\Big(G(z)+G(1-z)-G\Big) + n_f\,\Phi_g^q(z)\Big(2Q(z)-G\Big)\right], \quad (2)$$

where $\Phi_A^B(z)$ denote the DGLAP [6] splitting functions, $\alpha_s = 2\pi/4N_c\beta_0(\ell+y+\lambda)$ is the one-loop coupling constant of QCD [13] and

$$\ell = (1/x)\ ,\ y = \ln\left(k_\perp/Q_0\right)\ ,\ \lambda = \ln\left(Q_0/\Lambda_{_{\rm QCD}}\right),$$

(Q_0 being the collinear cut-off parameter), and where

$$G \equiv G(1) = xD_G^h(x, E\Theta_0, Q_0),$$
$$Q \equiv Q(1) = xD_Q^h(x, E\Theta_0, Q_0).$$

At small $x \ll z$, the fragmentation functions behave as

$$B(z) \approx \rho_B^h\left(\ln\frac{z}{x}, \ln\frac{zE\Theta_0}{Q_0}\right) = \rho_B^h\left(\ln z + \ell, y\right),$$

ρ_B^h being a slowly varying function of two logarithmic variables $\ln(z/x)$ and y that describes the "hump-backed" plateau [1]. In order to better account for recoil effects, the strategy followed in this Letter is to perform Taylor expansions (first advocated for in [7]) of the non-singular parts of the integrands in (1,2) in powers of $\ln z$ and $\ln(1-z)$, both considered small with respect to ℓ in the hard splitting region $z \sim 1-z = \mathcal{O}(1)$

$$B(z) = B(1) + B_\ell(1)\ln z + \mathcal{O}\left(\ln^2 z\right)\ ;\ z \leftrightarrow 1-z\,. \quad (3)$$

Each ℓ-derivative giving an extra $\sqrt{\alpha_s}$ factor (see [5]), the terms $B_\ell(1)\ln z$ and $B_\ell(1)\ln(1-z)$ yield NMLLA corrections to the solutions of (2). From (3) and the expressions of the DGLAP splitting functions, one gets after some algebra ($\gamma_0^2 = 2N_c\alpha_s/\pi$) [8]

$$Q(\ell,y) = \delta(\ell) + \frac{C_F}{N_c}\int_0^\ell \mathrm{d}\ell'\int_0^y \mathrm{d}y'\gamma_0^2(\ell'+y') \times \Big[1 - \tilde a_1\delta(\ell'-\ell) + \tilde a_2\delta(\ell'-\ell)\psi_\ell(\ell',y')\Big]G(\ell',y'), \quad (4)$$

$$G(\ell,y) = \delta(\ell) + \int_0^\ell \mathrm{d}\ell'\int_0^y \mathrm{d}y'\gamma_0^2(\ell'+y') \times \Big[1 - a_1\delta(\ell'-\ell) + a_2\delta(\ell'-\ell)\psi_\ell(\ell',y')\Big]G(\ell',y'). \quad (5)$$

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with $\psi_\ell(\ell,y) = G_\ell(\ell,y)/G(\ell,y)$. The MLLA coefficients $\tilde{a}_1 = 3/4$ and $a_1 \approx 0.935$ are computed in [5] while at NMLLA, we get [14]:

$$\tilde{a}_2 = \frac{7}{8} + \frac{C_F}{N_c}\left(\frac{5}{8} - \frac{\pi^2}{6}\right) \approx 0.42, \tag{6}$$

$$a_2 = \frac{67}{36} - \frac{\pi^2}{6} - \frac{13}{18}\frac{n_f T_R}{N_c}\frac{C_F}{N_c} \approx 0.06\ . \tag{7}$$

Computing the NMLLA partonic distributions inside a quark and gluon jet, $Q(z)$ and $G(z)$, is the first step to determine the double differential spectrum $\mathrm{d}^2N/\mathrm{d}x\,\mathrm{d}\Theta$ of a hadron produced with energy xE and at angle Θ with respect to the jet axis identified with the direction of the energy flow (see [8]). As shown in [9], it is given by

$$\frac{\mathrm{d}^2 N}{\mathrm{d}x\,\mathrm{d}\ln\Theta} = \frac{\mathrm{d}}{\mathrm{d}\ln\Theta} F_{A_0}^h\left(x,\Theta,E,\Theta_0\right), \tag{8}$$

where $F_{A_0}^h$ is given by the convolution of two fragmentation functions

$$F_{A_0}^h \equiv \sum_A \int_x^1 \mathrm{d}u D_{A_0}^A\left(u, E\Theta_0, uE\Theta\right) D_A^h\left(\frac{x}{u}, uE\Theta, Q_0\right), \tag{9}$$

u being the energy fraction of the intermediate parton A. $D_{A_0}^A$ describes the probability to emit A with energy uE off the parton A_0 (which initiates the jet), taking into account the evolution of the jet between Θ_0 and Θ. D_A^h describes the probability to produce the hadron h off A with energy fraction x/u and transverse momentum $k_\perp \approx uE\Theta \geq Q_0$ (see Fig. 1). As discussed in [9], the convolution (9) is dominated by $u \sim 1$ and therefore $D_{A_0}^A(u, E\Theta_0, uE\Theta)$ is given by DGLAP evolution [6]. On the contrary, the distribution $\tilde{D}_A^h \equiv \frac{x}{u} D_A^h\left(\frac{x}{u}, uE\Theta, Q_0\right) = \tilde{D}_A^h(\ell + \ln u, y)$ at low $x \ll u$ reduces to the hump-backed plateau,

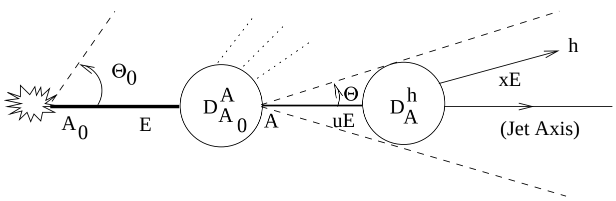

FIG. 1: Inclusive production of hadron h at angle Θ inside a high energy jet of total opening angle Θ_0 and energy E.

$$\tilde{D}_A^h(\ell + \ln u, y) \overset{x \ll u}{\approx} \rho_A^h(\ell + \ln u, Y_\Theta + \ln u), \tag{10}$$

with $Y_\Theta = \ell + y = \ln E\Theta/Q_0$. Performing the Taylor expansion of $\tilde{D}$ to the second order in $(\ln u)$ and plugging it into Eq. (9) leads to

$$\begin{aligned} xF_{A_0}^h \approx & \sum_A \int \mathrm{d}u\, u\, D_{A_0}^A(u, E\Theta_0, uE\Theta)\tilde{D}_A^h(\ell,y) \\ &+ \sum_A \int \mathrm{d}u\, u \ln u\, D_{A_0}^A(u, E\Theta_0, uE\Theta)\frac{\mathrm{d}\tilde{D}_A^h(\ell,y)}{\mathrm{d}\ell} \\ &+ \frac{1}{2}\sum_A \left[\int \mathrm{d}u\, u \ln^2 u D_{A_0}^A(u, E\Theta_0, uE\Theta)\right]\frac{\mathrm{d}^2\tilde{D}_A^h(\ell,y)}{\mathrm{d}\ell^2}. \end{aligned} \tag{11}$$

The first two terms in Eq. (11) correspond to the MLLA distribution calculated in [9] when $\tilde{D}_A^h$ is evaluated at NLO and its derivative at LO. NMLLA corrections arise from their respective calculation at NNLO and NLO, and, mainly in practice, from the third line, which is new. Indeed, since x/u is small, the inclusive spectrum $\tilde{D}_A^h(\ell,y)$ is the solution of the next-to-MLLA evolution equations (4) and (5). However, because of the smallness of the coefficient a_2 (see (7)), $G(\ell,y)$ shows no significant difference from MLLA to NMLLA. As a consequence, we use the MLLA expression for G. It is determined here from a representation in terms of a single Mellin transform of confluent hypergeometric functions (see Eq. (24) of [10]), well suited for numerical studies [15]. The NMLLA quark distribution $Q(\ell,y)$ can then be deduced from $G(\ell,y)$ using (4) and (5), which yields

$$\begin{aligned} Q(\ell,y) = & \frac{C_F}{N_c}\Big[G(\ell,y) + \Big(a_1 - \tilde{a}_1\Big)G_\ell(\ell,y) \\ &+ \Big(a_1\Big(a_1 - \tilde{a}_1\Big) + \tilde{a}_2 - a_2\Big) G_{\ell\ell}(\ell,y)\Big] + \mathcal{O}(\gamma_0^2). \end{aligned} \tag{12}$$

The functions F_g^h and F_q^h are related to the gluon distribution *via* the color currents $\langle C\rangle_{g,q}$ defined as:

$$xF_{g,q}^h = \frac{\langle C\rangle_{g,q}}{N_c}\, G(\ell,y). \tag{13}$$

$\langle C\rangle_{g,q}$ can be seen as the average color charge carried by the parton A due to the DGLAP evolution from A_0 to A. Introducing the first and second logarithmic derivatives of $\tilde{D}_A^h$,

$$\psi_{A,\ell}(\ell,y) = \frac{1}{\tilde{D}_A^h(\ell,y)}\frac{\mathrm{d}\tilde{D}_A(\ell,y)}{\mathrm{d}\ell} = \mathcal{O}(\sqrt{\alpha_s}),$$

$$(\psi_{A,\ell}^2 + \psi_{A,\ell\ell})(\ell,y) = \frac{1}{\tilde{D}_A^h(\ell,y)}\frac{\mathrm{d}^2\tilde{D}_A(\ell,y)}{\mathrm{d}\ell^2} = \mathcal{O}(\alpha_s),$$

Eq. (11) can now be written as

$$\begin{aligned} xF_{A_0}^h \approx & \sum_A \Big[\langle u\rangle_{A_0}^A + \langle u\ln u\rangle_{A_0}^A \psi_{A,\ell}(\ell,y) \\ &+ \frac{1}{2}\langle u \ln^2 u\rangle_{A_0}^A(\psi_{A,\ell}^2 + \psi_{A,\ell\ell})(\ell,y)\Big]\ \tilde{D}_A^h, \end{aligned} \tag{14}$$

with the notation

$$\langle u \ln^i u\rangle_{A_0}^A \equiv \int_0^1 \mathrm{d}u\ (u\ \ln^i u)\ D_{A_0}^A\left(u, E\Theta_0, uE\Theta\right)$$

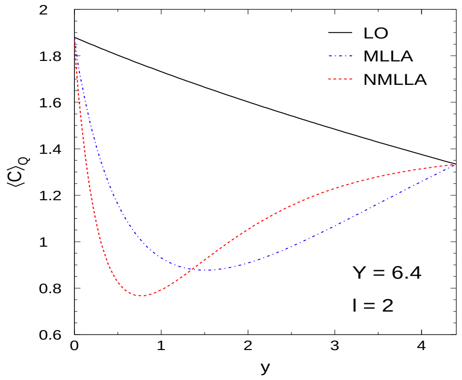

FIG. 2: The color current of a quark jet with $Y_{\Theta_0} = 6.4$ as a function of y at fixed $\ell = 2$.

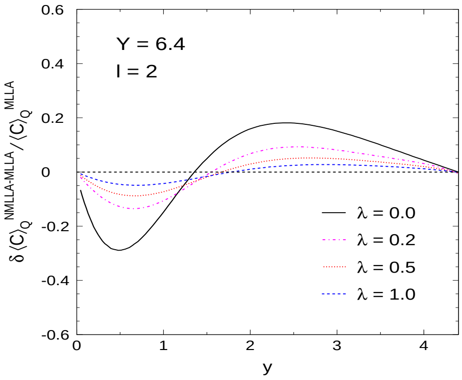

FIG. 3: NMLLA corrections to the color current of a quark jet with $Y_{\Theta_0} = 6.4$ and $\ell = 2$ for various values of λ.

$$\approx \int_0^1 \mathrm{d}u \; (u \; \ln^i u) \; D_{A_0}^A (u, E\Theta_0, E\Theta) \,. \qquad (15)$$

The scaling violation of the DGLAP fragmentation function neglected in the last approximation is a $\mathcal{O}(\alpha_s)$ correction to $\langle u \rangle$. It however never exceeds 5% [8] of the leading term and is thus neglected in the following. Using (13), the MLLA and NMLLA contributions to the leading color current of the parton $A_0 = g, q$ read

$$\begin{aligned} \delta\langle C\rangle_{A_0}^{\mathrm{MLLA-LO}} &= N_c \; \langle u \ln u \rangle_{A_0}^g \; \psi_{g,\ell} + \; C_F \; \langle u \ln u \rangle_{A_0}^q \; \psi_{q,\ell}, \\ \delta\langle C\rangle_{A_0}^{\mathrm{NMLLA-MLLA}} &= N_c \; \langle u \ln^2 u \rangle_{A_0}^g \; (\psi_{g,\ell}^2 + \psi_{g,\ell\ell}) \\ &\quad + C_F \; \langle u \ln^2 u \rangle_{A_0}^q \; (\psi_{q,\ell}^2 + \psi_{q,\ell\ell}). \end{aligned} \qquad (16)$$

The MLLA correction, $\mathcal{O}\left(\sqrt{\alpha_s}\right)$, was determined in [9] and the NMLLA contribution, $\mathcal{O}\left(\alpha_s\right)$, to the average color current is new. The latter can be obtained from the Mellin moments of the DGLAP fragmentation functions

$$\mathcal{D}_{A_0}^A(j, \xi) = \int_0^1 \mathrm{d}u \, u^{j-1} D_{A_0}^A(u, \xi),$$

leading to

$$\langle u \ln^2 u \rangle_{A_0}^A = \frac{\mathrm{d}^2}{\mathrm{d}j^2} \mathcal{D}_{A_0}^A(j, \xi(E\Theta_0) - \xi(E\Theta)) \bigg|_{j=2} . \qquad (17)$$

Plugging (17) into (16), the NMLLA color currents for gluon and quark jets are determined analytically [8]. For illustrative purposes, the LO, MLLA, and NMLLA average color current of a quark jet with $Y_{\Theta_0} = 6.4$ – corresponding roughly to Tevatron energies – is plotted in Fig. 2 as a function of y, at fixed $\ell = 2$. As discussed in [9], the MLLA corrections to the LO color current are found to be large and negative. As expected, the correction $\mathcal{O}\left(\alpha_s\right)$ from MLLA to NMLLA proves much smaller; it is negative (positive) at small (large) y.

This calculation has also been extended beyond the limiting spectrum, $\lambda \neq 0$, to take into account hadronization effects in the production of "massive" hadrons, $m = \mathcal{O}\left(Q_0\right)$ [10]. The NMLLA (normalized) corrections to the MLLA result are displayed in Fig. 3 for different values $\lambda = 0, 0.5, 1$. It clearly indicates that the larger λ, the smaller the NMLLA corrections. In particular, they can be as large as 30% at the limiting spectrum ($\lambda = 0$) but no more than 10% for $\lambda = 0.5$. This is not surprising since $\lambda \neq 0$ ($Q_0 \neq \Lambda_{\mathrm{QCD}}$) reduces the parton emission in the infrared sector and, thus, higher-order corrections.

The double differential spectrum $\mathrm{d}^2N/\mathrm{d}y\,\mathrm{d}\ell$, Eq. (8), can now be determined from the NMLLA color currents (16) using the MLLA quark and gluon distributions Integrating it over ℓ leads to the single inclusive y-distribution (or $k_\perp$-distribution) of hadrons inside a quark or a gluon jet:

$$\left(\frac{\mathrm{d}N}{\mathrm{d}y}\right)_{g,q} = \left(k_\perp \frac{\mathrm{d}N}{\mathrm{d}k_\perp}\right)_{g,q} = \int_{\ell_{\min}}^{Y_{\Theta_0}-y} \mathrm{d}\ell \, \left(\frac{\mathrm{d}^2N}{\mathrm{d}\ell\,\mathrm{d}y}\right)_{g,q} . \qquad (18)$$

The MLLA framework does not specify down to which values of ℓ (up to which values of x) the double differential spectrum $\mathrm{d}^2N/\mathrm{d}y\,\mathrm{d}\ell$ should be integrated over. Since $\mathrm{d}^2N/\mathrm{d}y\,\mathrm{d}\ell$ becomes negative (non-physical) at small values of ℓ (see e.g. [9]), we chose the lower bound $\ell_{\min}$ so as to guarantee the positiveness of $\mathrm{d}^2N/\mathrm{d}y\,\mathrm{d}\ell$ over the whole $\ell_{\min} \leq \ell \leq Y_{\Theta_0}$ range (in practice, $\ell_{\min}^g \sim 1$ and $\ell_{\min}^q \sim 2$).

Having successfully computed the single $k_\perp$-spectra including NMLLA corrections, we now compare the result with existing data. The CDF collaboration at the Tevatron recently reported on preliminary measurements over a wide range of jet hardness, $Q = E\Theta_0$, in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV [12]. CDF data, including systematic errors, are plotted in Fig. 4 together with the MLLA predictions of [9] and the present NMLLA calculations, both at the limiting spectrum ($\lambda = 0$) and taking $\Lambda_{\mathrm{QCD}} = 250$ MeV; the experimental distributions suffering from large normalization errors, data and theory are normalized to the same bin, $\ln(k_\perp/1\,\mathrm{GeV}) = -0.1$. The agreement between the CDF results and the NMLLA dis-

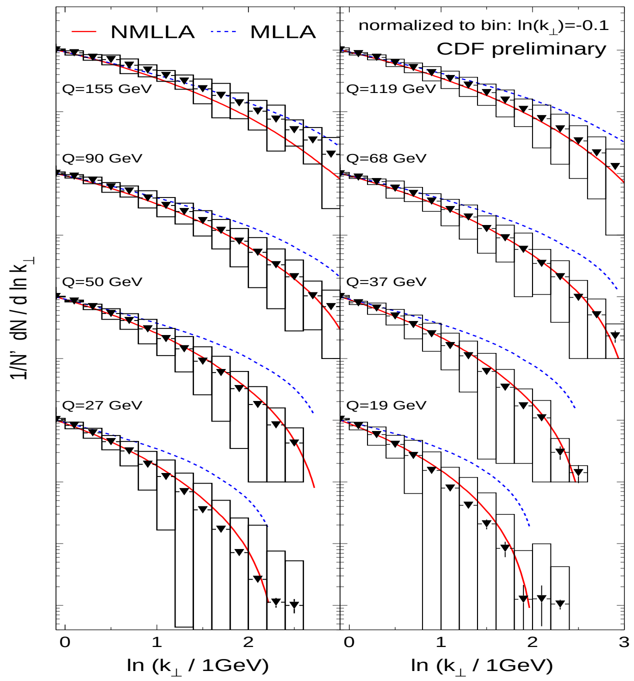

FIG. 4: CDF preliminary results for the inclusive $k_\perp$ distribution at various hardness Q in comparison to MLLA and NMLLA predictions at the limiting spectrum; the boxes are the systematic errors (their lower limits at large $k_\perp$ are cut for the sake of clarity).

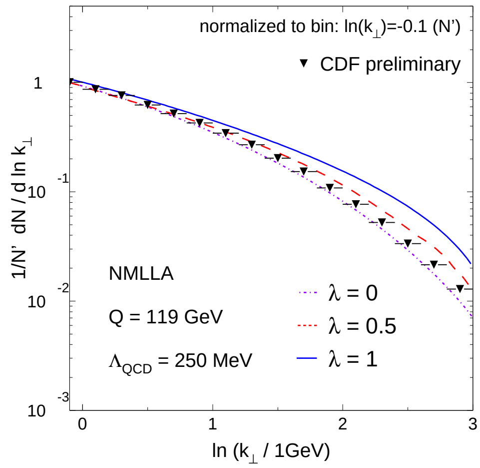

FIG. 5: CDF preliminary results ($Q = 119$ GeV) for inclusive $k_\perp$ distribution compared with NMLLA predictions beyond the limiting spectrum.

tributions over the whole $k_\perp$-range is particularly good. In contrast, the MLLA predictions prove reliable in a much smaller $k_\perp$ interval. At fixed jet hardness (and thus Y_{Θ_0}), NMLLA calculations prove accordingly trustable in a much larger x interval.

Despite this encouraging agreement with data, the present calculation still suffers from various theoretical uncertainties, discussed in detail in [8]. Among them, the variation of $\Lambda_{_{\rm QCD}}$ – giving NMLLA corrections – from the default value $\Lambda_{_{\rm QCD}} = 250$ MeV to 150 MeV and 400 MeV affects the normalized $k_\perp$-distributions by roughly 20% in the largest $\ln(k_\perp/1\ \text{GeV}) = 3$ GeV-bin at $Q = 100$ GeV. Also, cutting the integral (18) at small values of ℓ is somewhat arbitrary. However, we checked that changing $\ell^g_{\min}$ from 1 to 1.5 modifies the NMLLA spectra at large $k_\perp$ by $\sim 20\%$ only [16]. Finally, the $k_\perp$-distribution is determined with respect to the jet energy flow from 2-particle correlations (which includes a summation over secondary hadrons), while experimentally the jet axis is determined exclusively from *all* particles inside the jet. The question of the matching of these two definitions at $\mathcal{O}(\alpha_s)$ accuracy goes beyond the scope of this Letter.

The NMLLA $k_\perp$-spectrum has also been calculated beyond the limiting spectrum, as illustrated in Fig. 5. However, the best description of CDF preliminary data is reached at the limiting spectrum, or at least for small values of $\lambda \lesssim 0.5$, which is not too surprising since these inclusive measurements mostly involve pions. Identifying produced hadrons would offer the interesting possibility to check a dependence of the shape of $k_\perp$-distributions on the hadron species, such as the one predicted in Fig. 5.

To summarize, single inclusive $k_\perp$-spectra inside a jet are determined including higher-order $\mathcal{O}(\alpha_s)$ (i.e. NMLLA) corrections from the Taylor expansion of the MLLA evolution equations and beyond the limiting spectrum, $\lambda \neq 0$. The agreement between NMLLA predictions and CDF preliminary data in $p\bar{p}$ collisions at the Tevatron is very good, indicating very small overall non-perturbative corrections. The MLLA evolution equations for inclusive enough variables prove once more (see e.g. [6]) to include reliable information at a higher precision than the one at which they have been deduced.

Acknowledgments: We gratefully acknowledge enlightening discussions with Yu.L. Dokshitzer, I.M. Dremin, S. Jindariani (CDF), W. Ochs and M. Rubin.

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[13] A 2-loop evaluation of the splitting functions and α_s would not fit into the present logic of a systematic expansion in powers of $\sqrt{\alpha_s}$ (see [8]).
[14] Assuming $Q/G = C_F/N_c$. We checked that $\mathcal{O}\left(\sqrt{\alpha_s}\right)$ and $\mathcal{O}\left(\alpha_s\right)$ corrections affect marginally these coefficients.
[15] It was also given in [5] a compact Mellin representation from which an analytic approximated expression was found using the steepest descent method [11].
[16] The effect of varying $\ell_{\min}$ is more dramatic at MLLA.